How many infinities are there?

Pye Aung

Since the dawn of civilization, we have become increasingly better at counting. However, when we attempt to count beyond finite quantities, we run into a somewhat bizarre world that often seems counter-intuitive. We will talk about some motivation behind some counting techniques, and “different kinds” of infinities, as to whether it is countable or uncountable. We will also briefly discuss cardinal numbers, especially $\aleph_0$ and $\aleph_1$, and different examples from various fields of mathematics, such as calculus and probability theory, where countability plays an important role. Along the way, we will see the famous Cantor’s diagonal argument, the axiom of choice, Cantor-Bernstein-Schroeder Theorem, and Continuum Hypothesis. Time permitting, we may also briefly discuss, without proofs, about two cardinals $p$ and $t$, as seen in E.K. van Douwen’s Integers and Topology, which were recently proven to be equal to each other. The talk is accessible to all undergraduate students, although some calculus is necessary to appreciate certain examples.